

Hadronic Light by Light & Muon g-2



M.J. Ramsey-Musolf

Wisconsin-Madison



NPAC

Theoretical Nuclear, Particle, Astrophysics & Cosmology

<http://www.physics.wisc.edu/groups/particle-theory/>

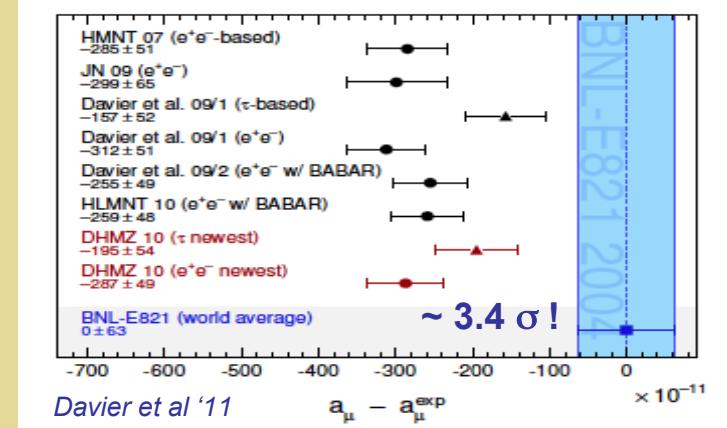
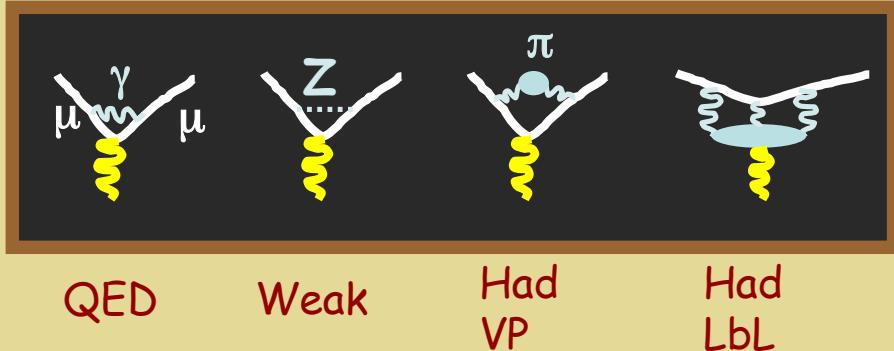
Confinement X, October 2012

Outline

- I. *Intro & Motivation: SM & Beyond*
- II. *Hadronic Light-by-Light: Review & Status*
- III. *Charged Pion Loops revisited*
- IV. *Summary and Outlook*

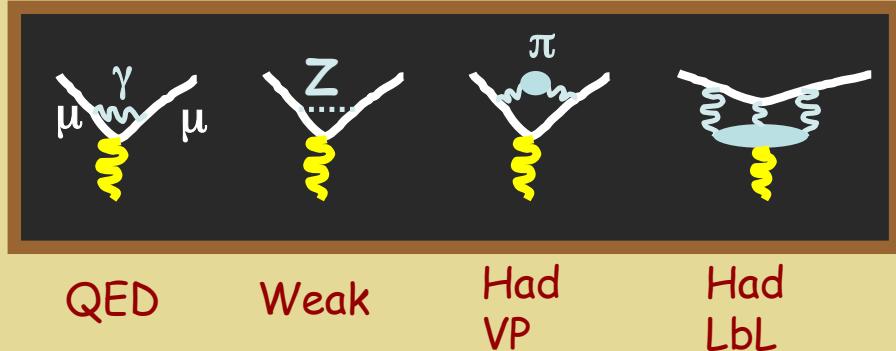
Intro & Motivation: SM & Beyond

Muon Anomalous Magnetic Moment

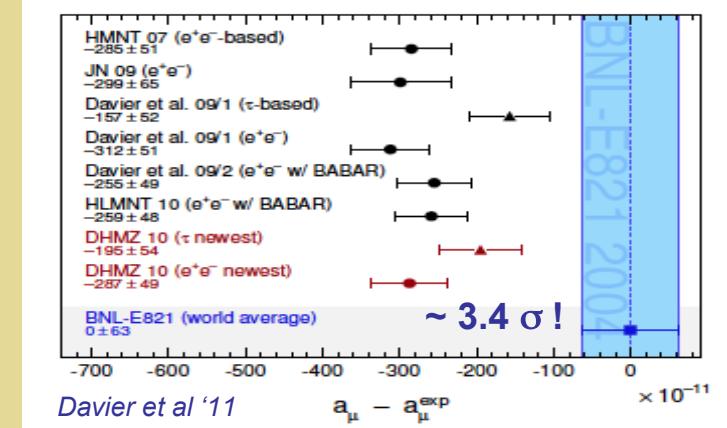
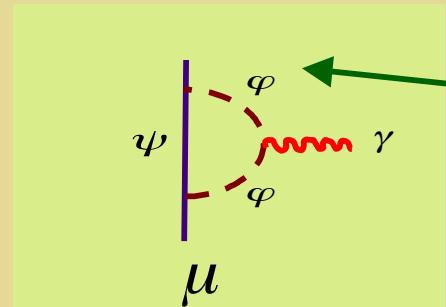


SM Loops

Muon Anomalous Magnetic Moment

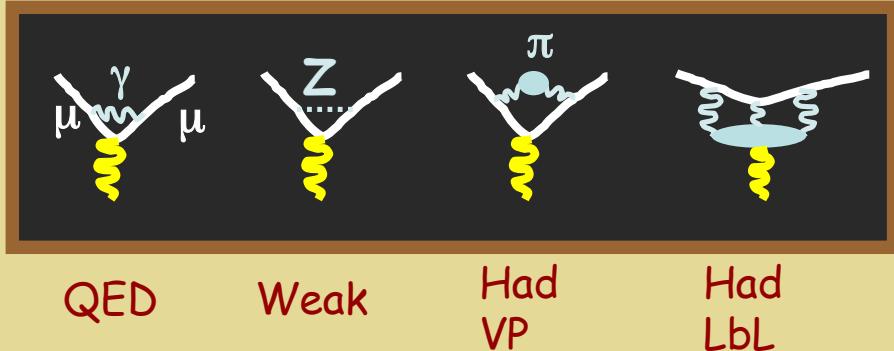


SM Loops

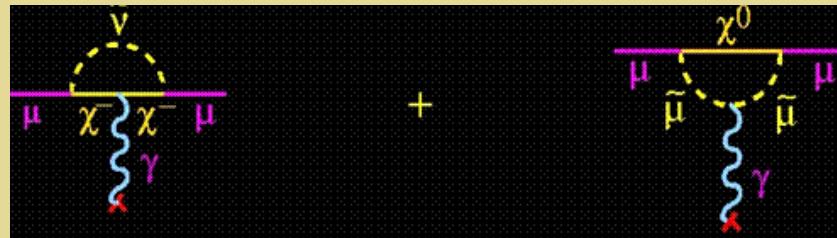


- Smuon (SUSY)
- Heavy Z'
- Leptoquark
- Extended scalar sector
- “Dark photon”

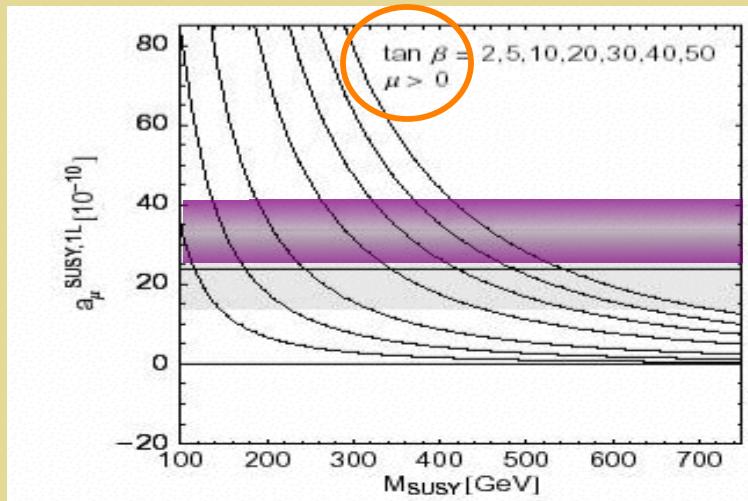
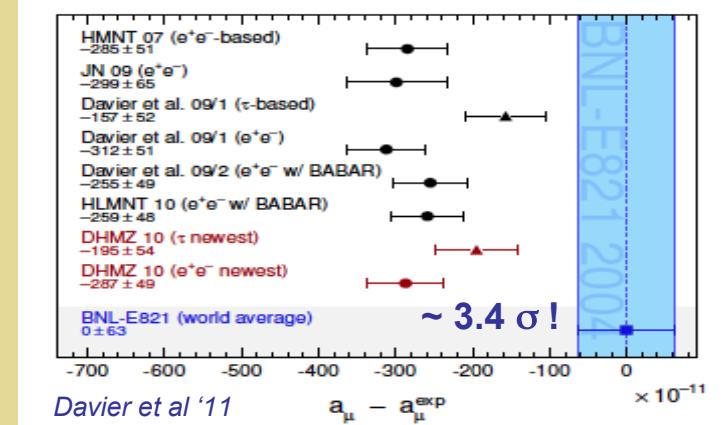
Muon Anomalous Magnetic Moment



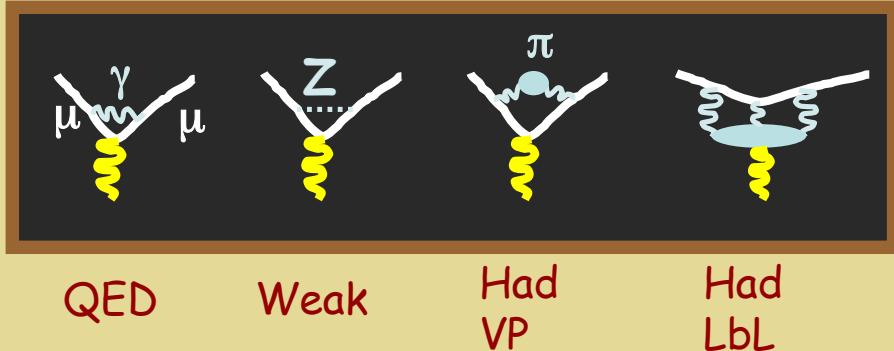
SM Loops



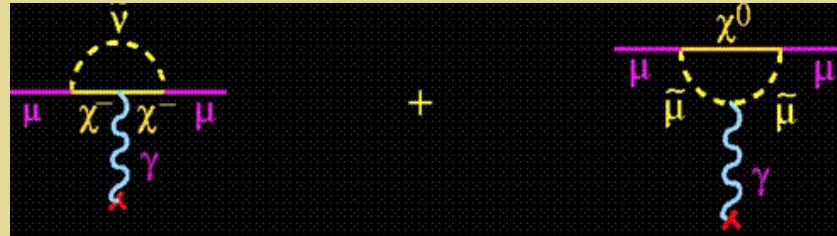
SUSY Loops



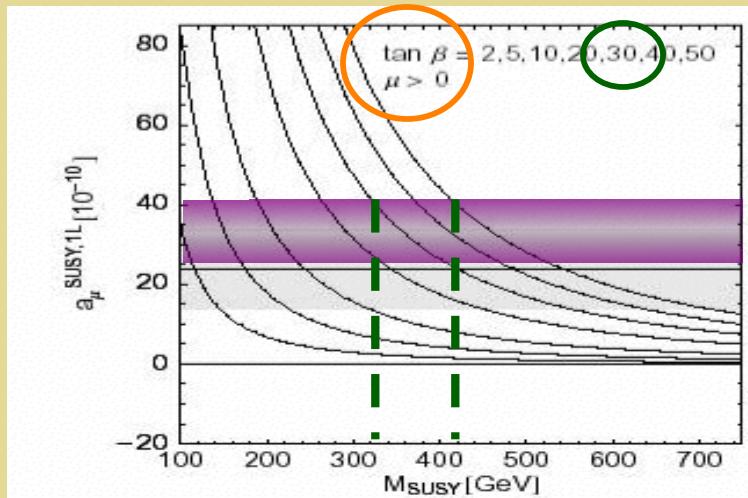
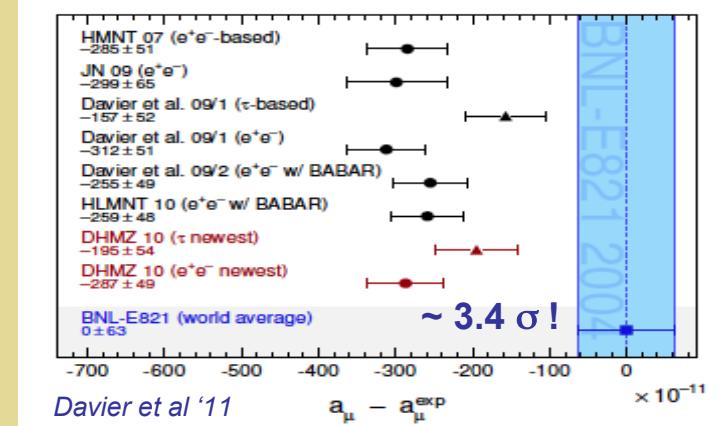
Muon Anomalous Magnetic Moment



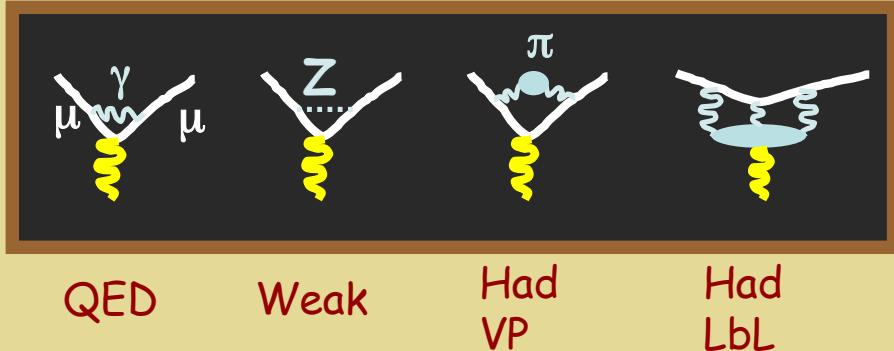
SM Loops



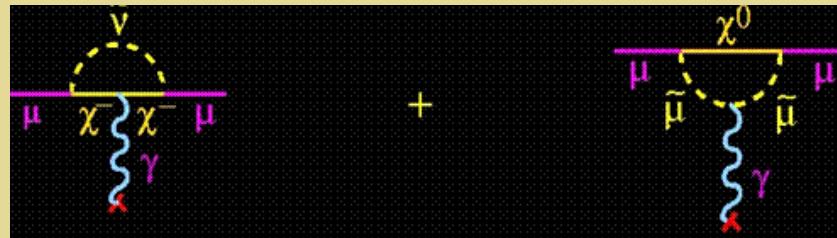
SUSY Loops



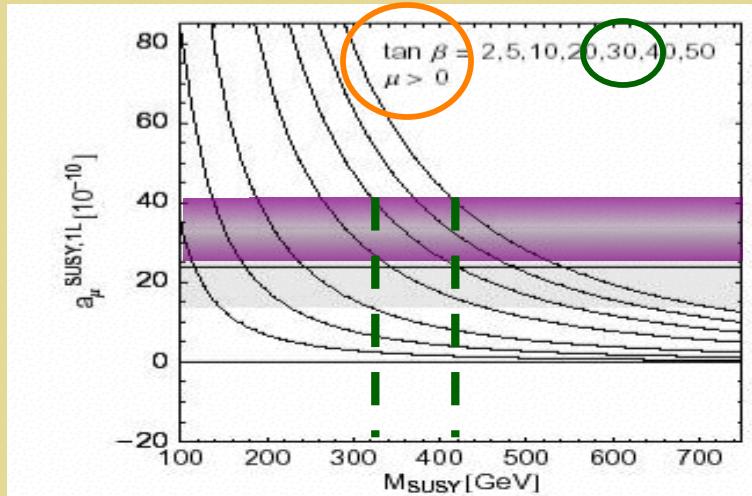
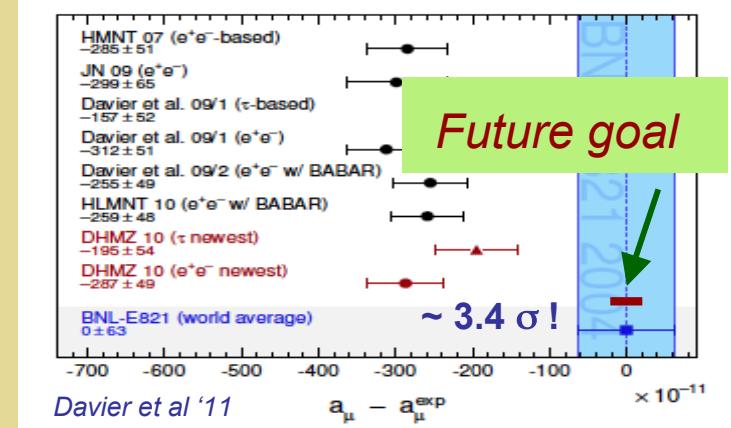
Muon Anomalous Magnetic Moment



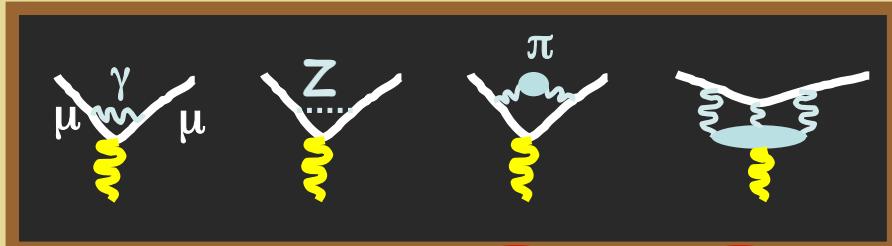
SM Loops



SUSY Loops



Muon Anomalous Magnetic Moment



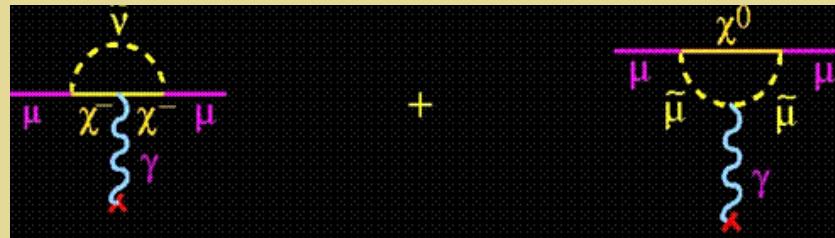
QED

Weak

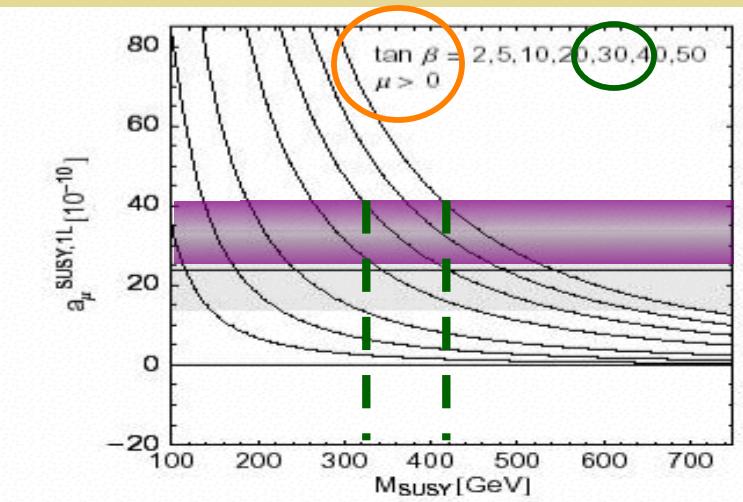
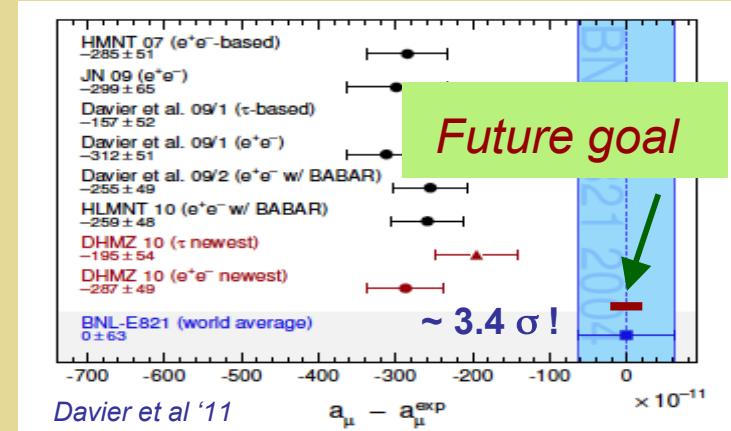
Had
VP

Had
LbL

SM Loops



SUSY Loops



Theory Error Budget

$$a_\mu(EW) = 154 (2) \times 10^{-11}$$

$$a_\mu(HVP-LO) = 6894 (40) \times 10^{-11}$$

$$a_\mu(HVP-NLO) = -98 (1) \times 10^{-11}$$

$$\begin{aligned} a_\mu(HLBL) &= 116 (39) \times 10^{-11} \\ &105 (26) \times 10^{-11} \end{aligned}$$

$$\delta a_\mu^{TH} = {}^{+}_{-} 48 \times 10^{-11}$$

$$\delta a_\mu^{EXP} = {}^{+}_{-} 63 \times 10^{-11} \quad BNL E821$$

Theory Error Budget

$$a_\mu(EW) = 154 (2) \times 10^{-11}$$

$$a_\mu(HVP-LO) = 6894 (40) \times 10^{-11}$$

$$a_\mu(HVP-NLO) = -98 (1) \times 10^{-11}$$

$$\begin{aligned} a_\mu(HLBL) &= 116 (39) \times 10^{-11} \\ &105 (26) \times 10^{-11} \end{aligned}$$

$$\delta a_\mu^{TH} = {}^{+}_{-} 48 \times 10^{-11}$$

$$\delta a_\mu^{EXP} = {}^{+}_{-} 63 \times 10^{-11} \quad BNL E821$$

$$\Delta a_\mu = a_\mu^{EXP} - a_\mu^{TH} = 316 (79) \times 10^{-11}$$

Theory Error Budget

$$a_\mu(EW) = 154 (2) \times 10^{-11}$$

$$a_\mu(HVP-LO) = 6894 (40) \times 10^{-11}$$

$$a_\mu(HVP-NLO) = -98 (1) \times 10^{-11}$$

$$\begin{aligned} a_\mu(HLBL) &= 116 (39) \times 10^{-11} \\ &105 (26) \times 10^{-11} \end{aligned}$$

$$\delta a_\mu^{TH} = {}^{+}_{-} 48 \times 10^{-11}$$

$$\begin{aligned} \delta a_\mu^{EXP} &= {}^{+}_{-} 63 \times 10^{-11} && BNL E821 \\ &{}^{+}_{-} 15 \times 10^{-11} && FNAL New g-2 \end{aligned}$$

$$\Delta a_\mu = a_\mu^{EXP} - a_\mu^{TH} = 316 (79) \times 10^{-11}$$

Theory Error Budget

$$a_\mu(EW) = 154 (2) \times 10^{-11}$$

$$a_\mu(HVP-LO) = 6894 (40) \times 10^{-11}$$

$$a_\mu(HVP-NLO) = -98 (1) \times 10^{-11}$$

$$a_\mu(HLBL) = 116 (39) \times 10^{-11}$$
$$105 (26) \times 10^{-11}$$

Most challenging

$$\delta a_\mu^{TH} = {}^{+}_{-} 48 \times 10^{-11}$$

$$\delta a_\mu^{EXP} = {}^{+}_{-} 63 \times 10^{-11}$$

BNL E821

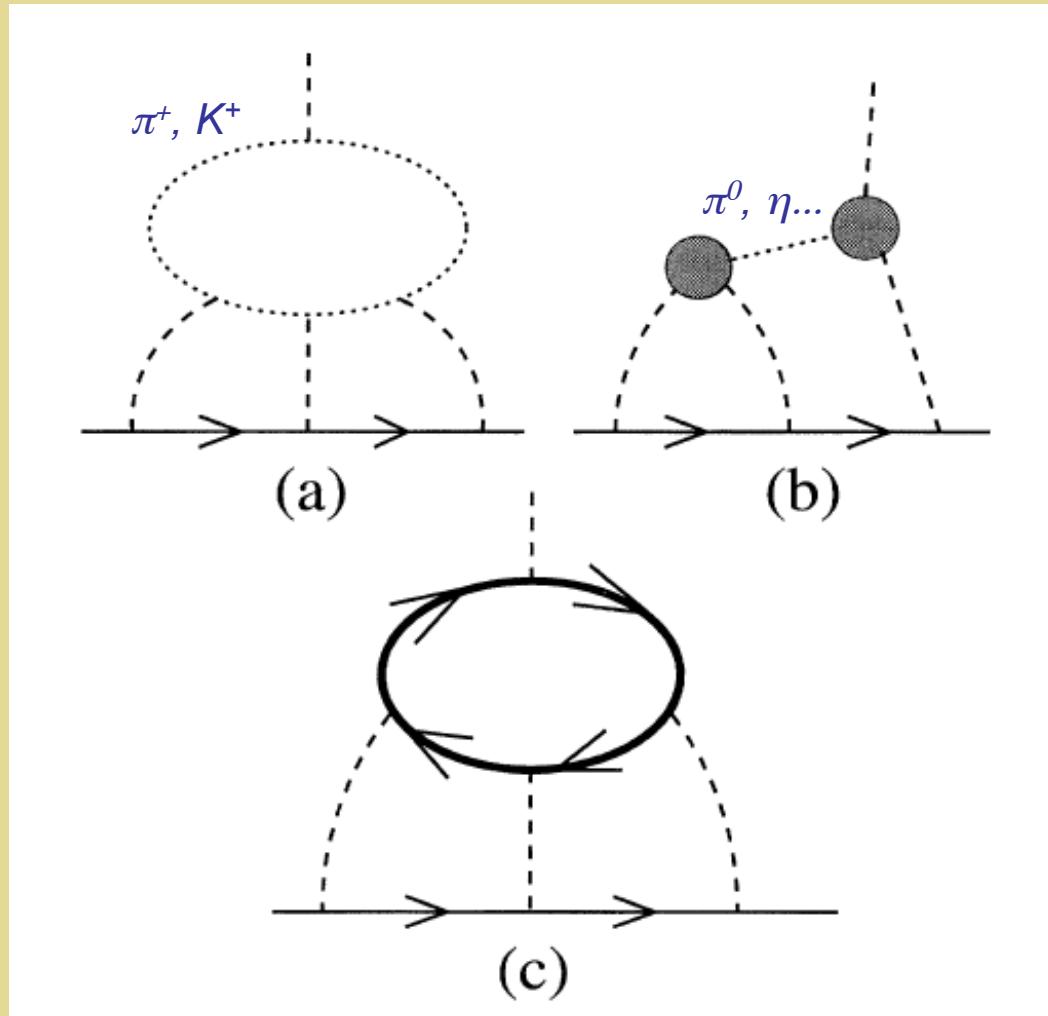
FNAL New g-2

$$\Delta a_\mu = a_\mu^{EXP} - a_\mu^{TH} = 316 (79) \times 10^{-11}$$

Hadronic Light-by-Light: Review & Status

HLBL Contributions

*Pseudoscalar
Loops*

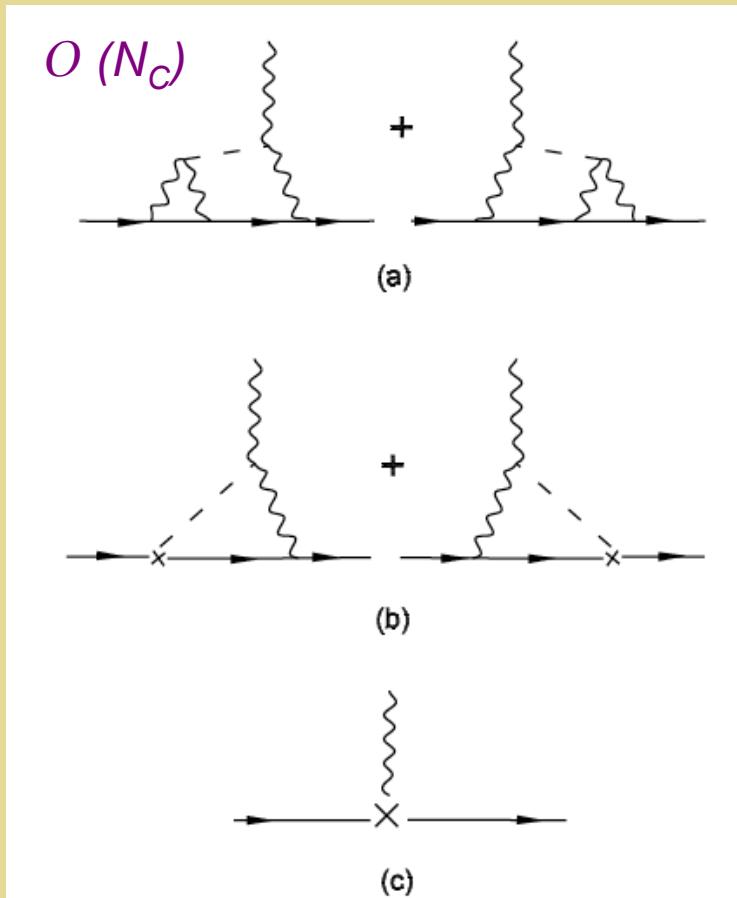


*Pseudoscalar
Poles*

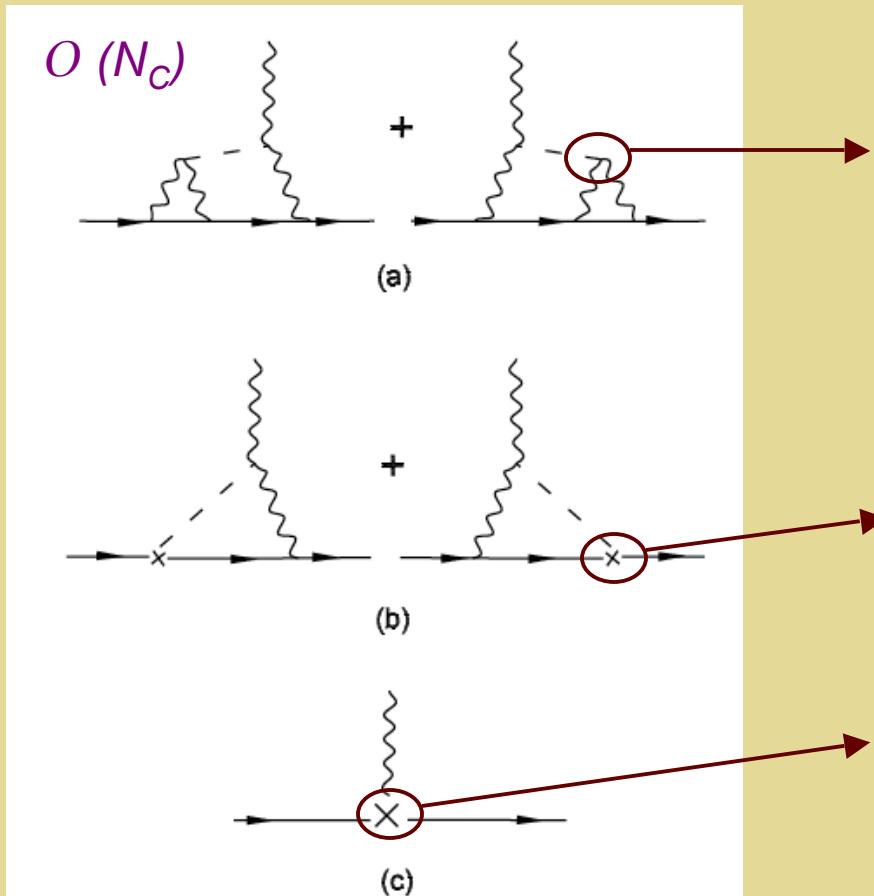
*Quark
Loops*

Hayakawa, Kinoshita, Sanda '95

Pseudoscalar Pole Contribution



Pseudoscalar Pole Contribution



\mathcal{L}_{WZW} : \ln^2 term \rightarrow

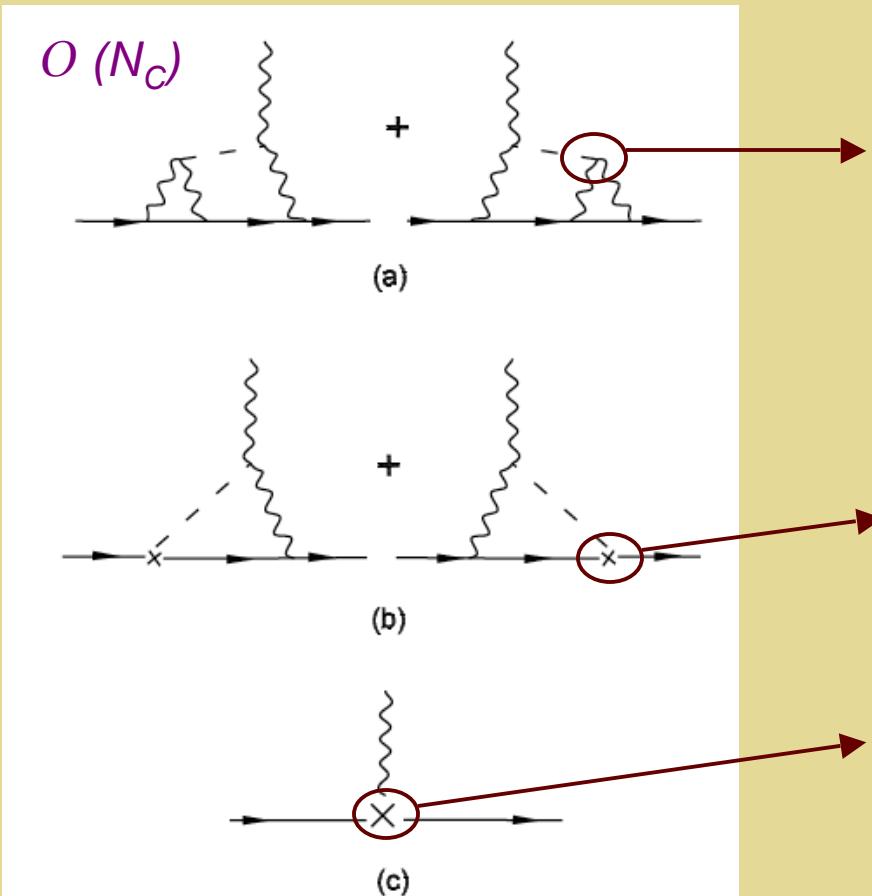
Sign error discovered
by Knecht et al

$\mathcal{P} \rightarrow \ell^+ \ell^-$: \ln term \rightarrow

Exp't (MRM, Wise)

Overall LEC \rightarrow
Models, lattice QCD...

Pseudoscalar Pole Contribution



\mathcal{L}_{WZW} : \ln^2 term \rightarrow

Sign error discovered
by Knecht et al

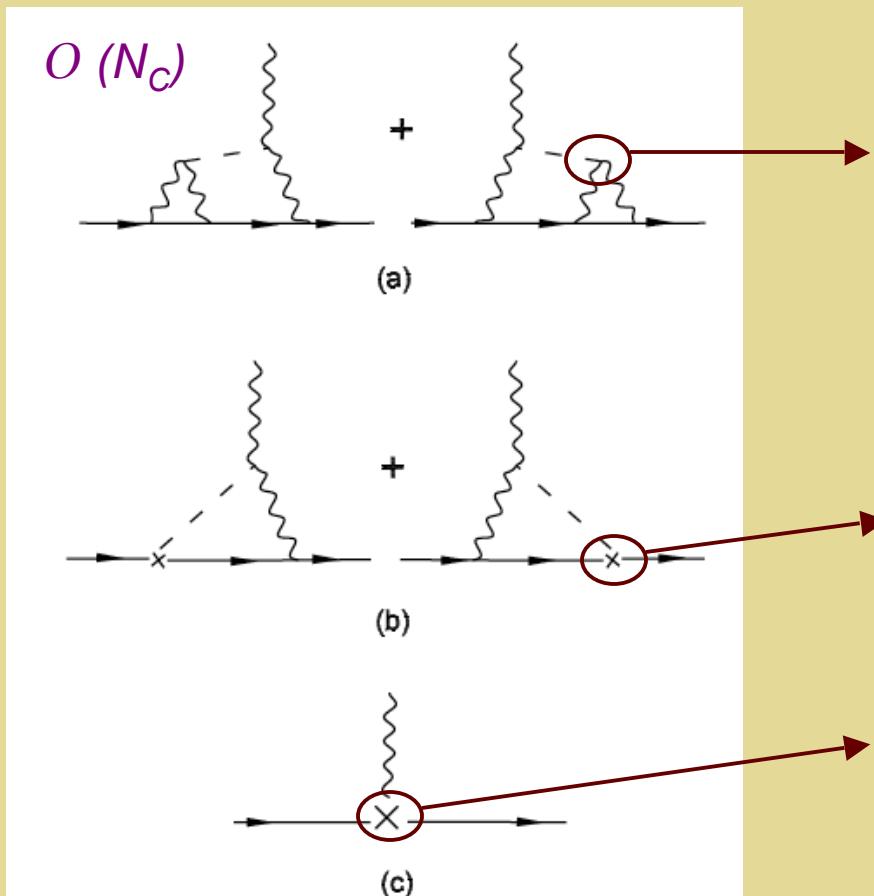
$\mathcal{P} \rightarrow \ell^+ \ell^-$: \ln term \rightarrow

Exp't (MRM, Wise)

Overall LEC \rightarrow
Models, lattice QCD...

$$a_\mu(\chi PT) = (57^{+50}_{-60} + 31 C) \times 10^{-11}$$

Pseudoscalar Pole Contribution



\mathcal{L}_{WZW} : \ln^2 term \rightarrow

Sign error discovered
by Knecht et al

$\mathcal{P} \rightarrow \ell^+ \ell^-$: \ln term \rightarrow

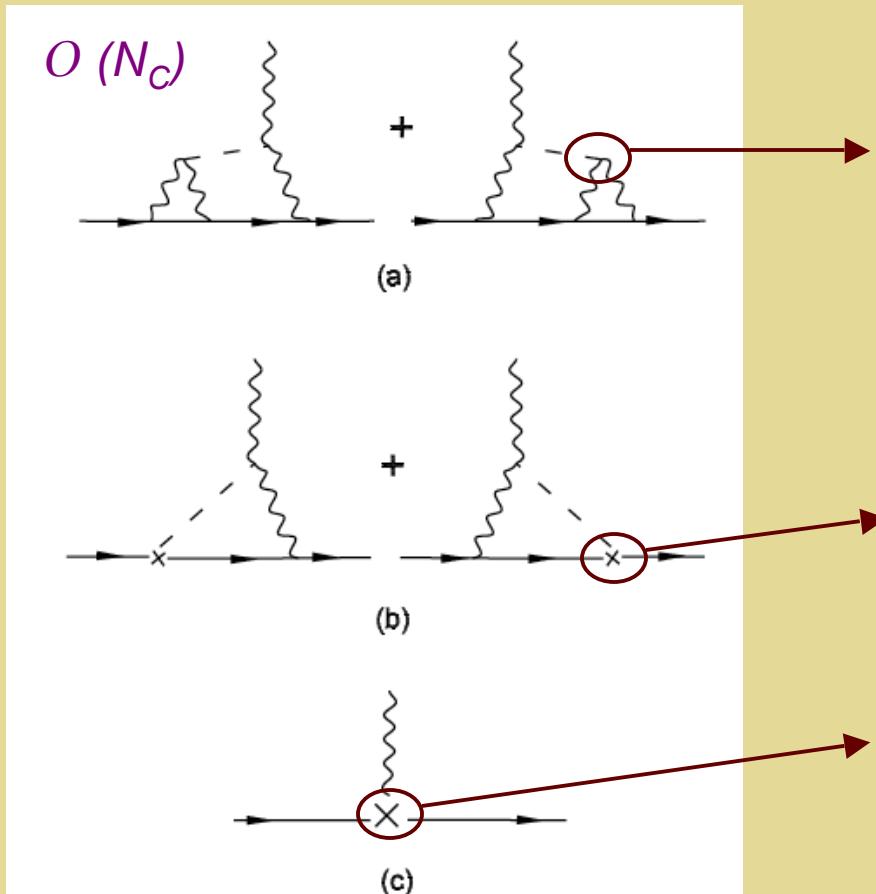
Exp't (MRM, Wise)

Overall LEC \rightarrow
Models, lattice QCD...

$$a_\mu(\chi PT) = (57^{+50}_{-60} + 31 C) \times 10^{-11}$$

Significantly reduced: KTeV '07

Pseudoscalar Pole Contribution



\mathcal{L}_{WZW} : \ln^2 term \rightarrow

Sign error discovered
by Knecht et al

$\mathcal{P} \rightarrow \ell^+ \ell^-$: \ln term \rightarrow

Exp't (MRM, Wise)

Overall LEC \rightarrow
Models, lattice QCD...

$$a_\mu(\chi PT) = (57^{+50}_{-60} + 31 C) \times 10^{-11}$$

Models: $C \sim 2$

$\sim 1\sigma$

E821: $C \sim 10$

Representative Models

- *Hidden Local Symmetry (HLS) [1]*
- *Extended NJL (ENJL)/VMD[1,2]*
- *Constituent Chiral Quark Model ($C\chi$ QM) [3]*
- *AdS/CFT [4]*
- *Dyson-Schwinger [5]*

[1] Hayakawa, Kinoshita, Sanda ‘95

[2] Bijnens, Pallante, Prades ‘96

[3] De Rafael ‘12; Boughezal & Melkinov ‘11

[4] Hong & Kim ‘09; Cappiello, Cata, D’Ambrosio ‘11

[5] Goeke, Fischer, Williams ‘11, ‘12

Representative Models

- Hidden Local Symmetry (HLS) [1]
- Extended NJL (ENJL)/VMD[1,2]
- Constituent Chiral Quark Model ($C\chi$ QM) [3]
- AdS/CFT [4]
- Dyson-Schwinger [5]

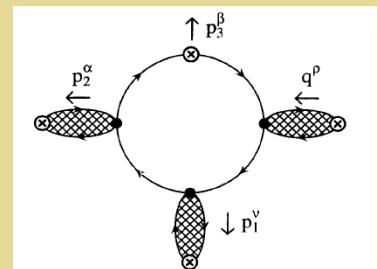
[1] Hayakawa, Kinoshita, Sanda '95

[2] Bijnens, Pallante, Prades '96

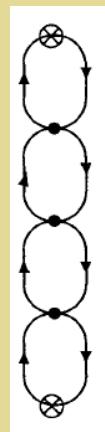
[3] De Rafael '12; Boughezal & Melkinov '11

[4] Hong & Kim '09; Cappiello, Cata, D'Ambrosio '11

[5] Goeke, Fischer, Williams '11, '12

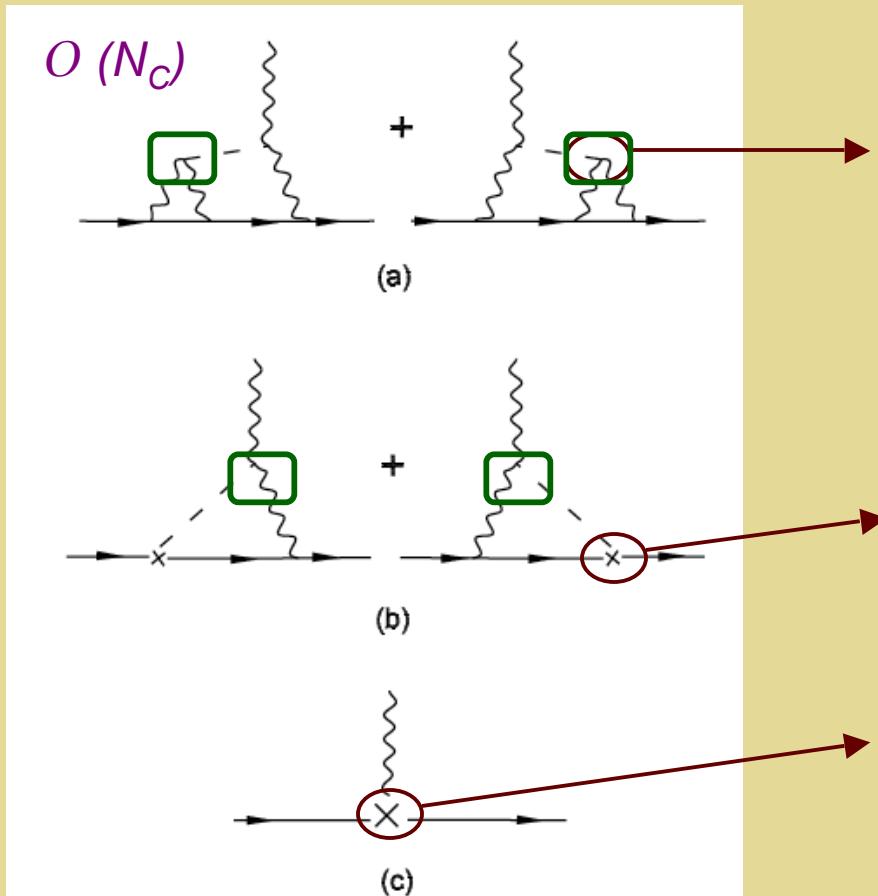


ρ pole



Short Distance Constraints

Vainshtein & Melnikov '04



\mathcal{L}_{WZW} : \ln^2 term \rightarrow

Sign error discovered
by Knecht et al

$\mathcal{P} \rightarrow \ell^+ \ell^-$: \ln term \rightarrow

Exp't (MRM, Wise)

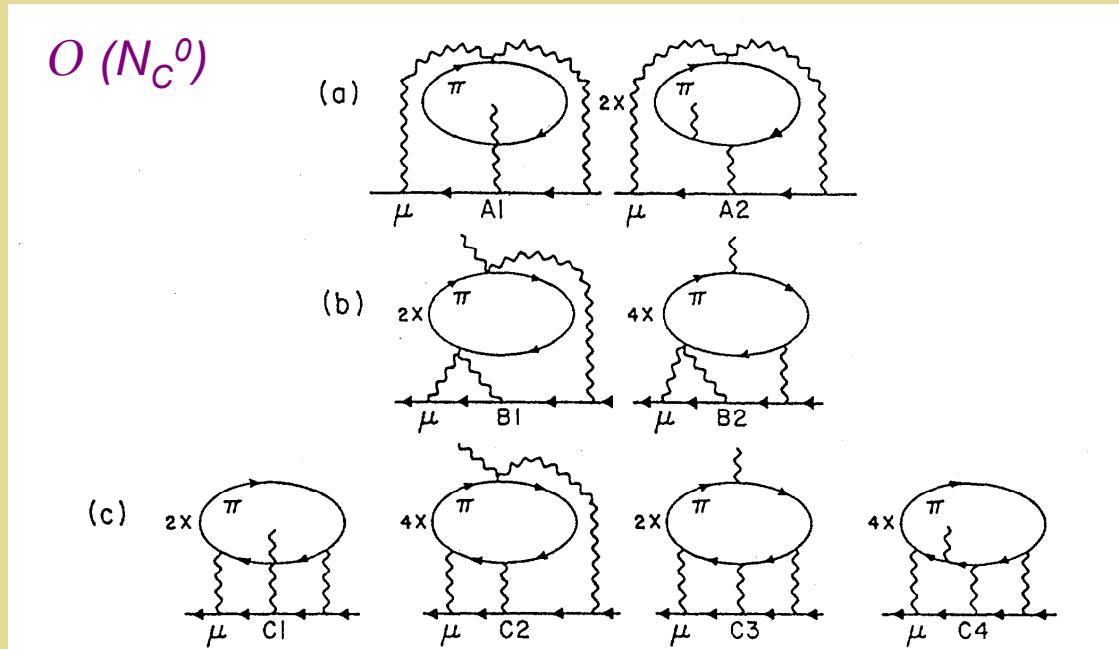
Overall LEC \rightarrow
Models, lattice QCD...

$$\Delta a_\mu(\text{OPE}) = 30 \times 10^{-11} \quad (\rightarrow C = +1)$$

Charged Pion Loops Revisited

Charged Pion Contribution

Kinoshita, Nizic, Okamoto '85 ; Hayakawa, Kinoshita, Sanda '95



$$\text{Point-like pions: } -0.0383 (19) (\alpha / \pi)^3 = -48 (2) \times 10^{-11}$$

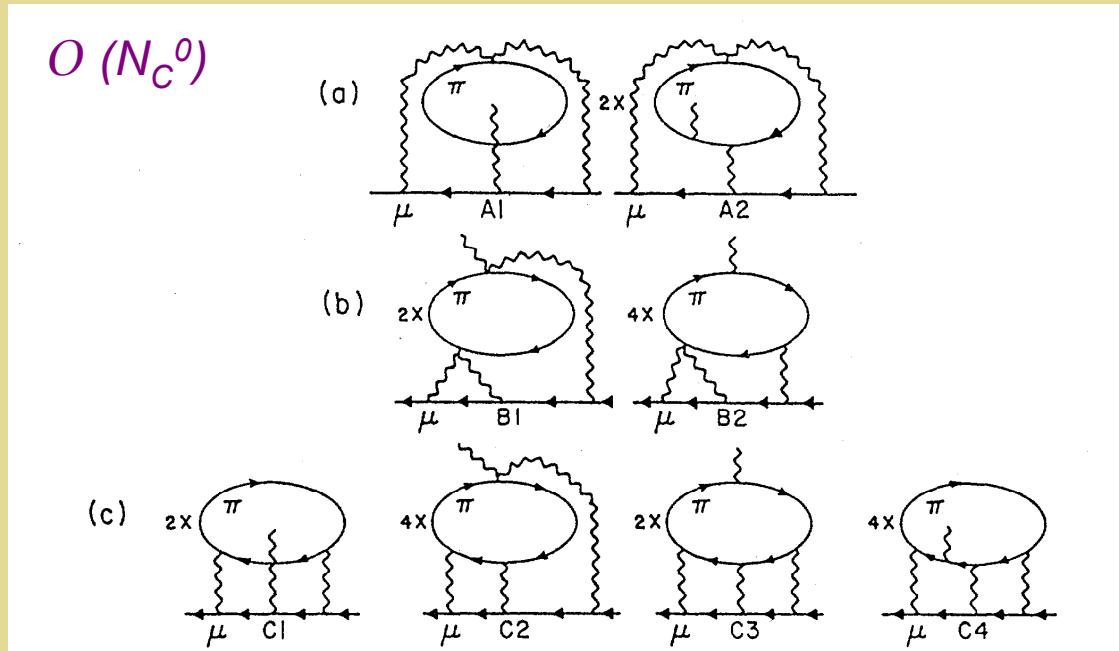
$$\text{Include } F_\pi(q^2): \quad -0.0125 (19) (\alpha / \pi)^3 = -16 (2) \times 10^{-11}$$

$$\text{"HLS":} \quad -0.00355 (12) (\alpha / \pi)^3 = -4.5 (0.2) \times 10^{-11}$$

$$ENJL: \quad -0.015 (4) (\alpha / \pi)^3 = -19 (5) \times 10^{-11}$$

Charged Pion Contribution

Kinoshita, Nizic, Okamoto '85 ; Hayakawa, Kinoshita, Sanda '95



Substantial
NLO impact

$$\text{Point-like pions: } -0.0383 (19) (\alpha / \pi)^3 = -48 (2) \times 10^{-11}$$

$$\text{Include } F_\pi (q^2): -0.0125 (19) (\alpha / \pi)^3 = -16 (2) \times 10^{-11}$$

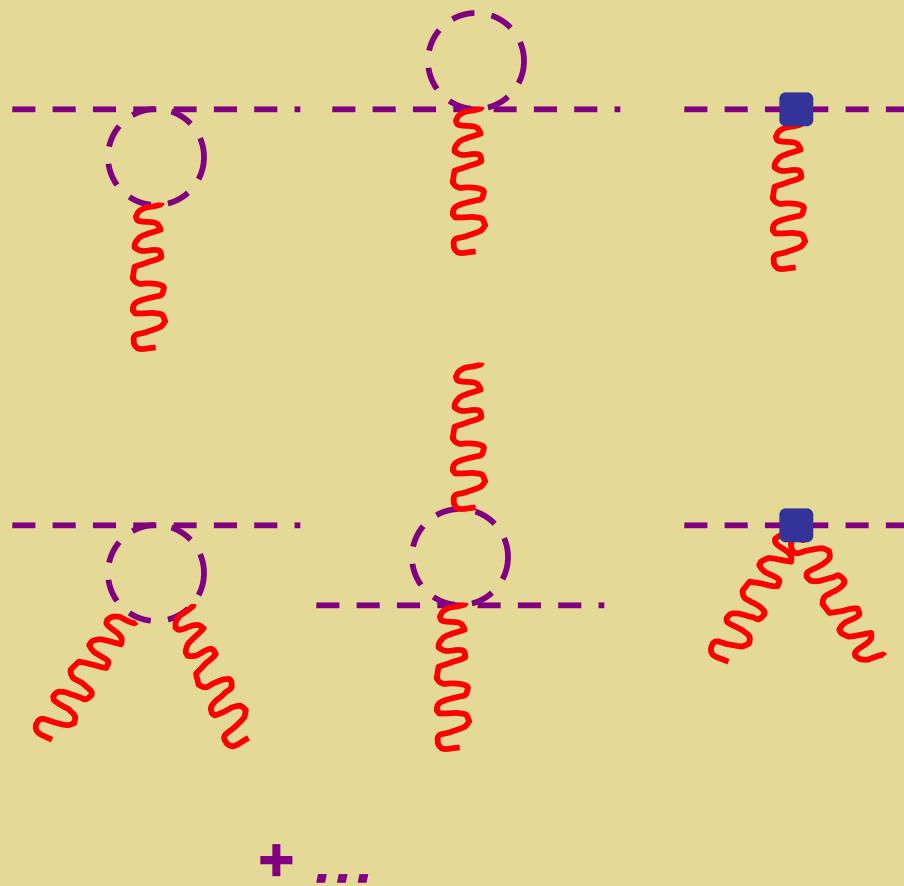
$$\text{"HLS": } -0.00355 (12) (\alpha / \pi)^3 = -4.5 (0.2) \times 10^{-11}$$

$$ENJL: -0.015 (4) (\alpha / \pi)^3 = -19 (5) \times 10^{-11}$$

Charged Pion Contribution: χ PT

Kevin Engel (Caltech), Hiren Patel (Wisconsin), MRM

Beyond leading order: subgraphs



Pion charge radius:
first non-trivial term in
expansion of $F_\pi(q^2)$

$O(p^4)$ LEC: α_9

Pion polarizability:
distinct physics from ff

$O(p^4)$ LEC: $\alpha_9 + \alpha_{10}$

Charged Pion Contribution: χ PT

Kevin Engel (Caltech), Hiren Patel (Wisconsin), MRM

Beyond leading order: embedding
subgraphs in full HLBL contribution

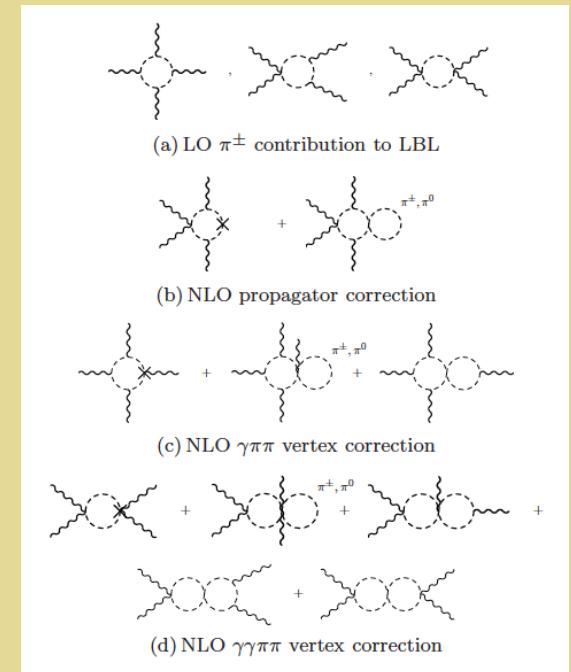
d=8 ops

Operator	1 loop χ PT	2 loop	VMD
$\mathcal{O}_1^{(8)}$	$1/9$	$\frac{m_\pi^2}{F_\pi^2} \frac{16}{3} (\alpha_9^r + \alpha_{10}^r)$	0
$\mathcal{O}_2^{(8)}$	$1/45$	0	0

n	1 loop	2 loop	VMD
1	$\frac{1}{45}$	$\frac{1}{3} \left\{ \frac{1}{9} (m_\pi r_\pi)^2 + \frac{4}{5} \left(\frac{m_\pi}{F_\pi} \right)^2 (\alpha_9^r + \alpha_{10}^r) \right\}$	$\frac{2}{9} \frac{m_\pi^2}{M_V^2}$
2	$\frac{2}{45}$	$\frac{1}{9} \left\{ \frac{1}{3} (m_\pi r_\pi)^2 + \frac{1}{2} \frac{m_\pi^2}{\Lambda_\chi^2} + \frac{44}{5} \left(\frac{m_\pi}{F_\pi} \right)^2 (\alpha_9^r + \alpha_{10}^r) \right\}$	$\frac{2}{9} \frac{m_\pi^2}{M_V^2}$
3	$\frac{2}{315}$	$\frac{1}{135} (m_\pi r_\pi)^2$	$\frac{2}{45} \frac{m_\pi^2}{M_V^2}$
4	$\frac{1}{189}$	$\frac{1}{135} (m_\pi r_\pi)^2$	$\frac{2}{45} \frac{m_\pi^2}{M_V^2}$
5	$\frac{1}{135}$	$\frac{4}{45} \left(\frac{m_\pi}{F_\pi} \right)^2 (\alpha_9^r + \alpha_{10}^r)$	0
6	$\frac{1}{315}$	0	0
7	$\frac{1}{945}$	0	0

d=10 ops

PRD 86:037502
(2012)



Charged Pion Contribution: χ PT

Kevin Engel (Caltech), Hiren Patel (Wisconsin), MRM

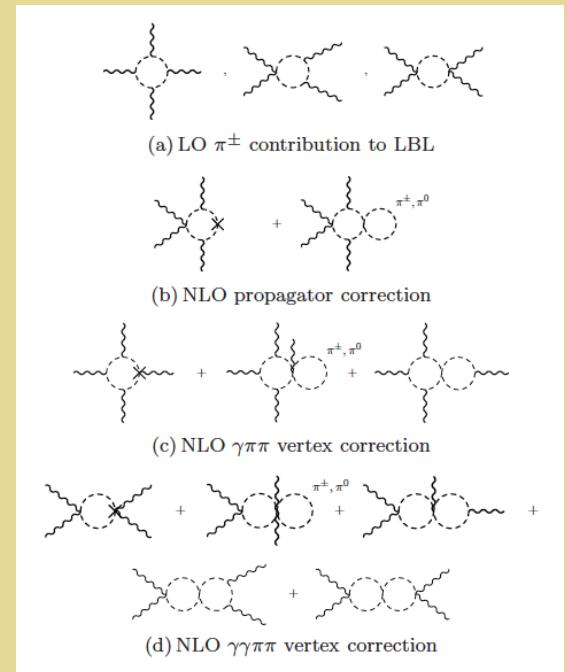
Beyond leading order: embedding
subgraphs in full HLBL contribution

Operator	1 loop χ PT	2 loop	VMD
$\mathcal{O}_1^{(8)}$	1/9	$\frac{\pi^2}{F_\pi^2} \frac{16}{3} (\alpha_9^r + \alpha_{10}^r)$	0
$\mathcal{O}_2^{(8)}$	1/45	0	0

LO:
suppressed

n	1 loop	2 loop	VMD
1	$\frac{1}{45}$	$\frac{1}{3} \left\{ \frac{1}{9} (m_\pi r_\pi)^2 + \frac{4}{5} \left(\frac{m_\pi}{F_\pi} \right)^2 (\alpha_9^r + \alpha_{10}^r) \right\}$	$\frac{2}{9} \frac{m_\pi^2}{M_V^2}$
2	$\frac{2}{45}$	$\frac{1}{9} \left\{ \frac{1}{3} (m_\pi r_\pi)^2 + \frac{1}{2} \frac{m_\pi^2}{\Lambda_\chi^2} + \frac{44}{5} \left(\frac{m_\pi}{F_\pi} \right)^2 (\alpha_9^r + \alpha_{10}^r) \right\}$	$\frac{2}{9} \frac{m_\pi^2}{M_V^2}$
3	$\frac{2}{315}$	$\frac{1}{135} (m_\pi r_\pi)^2$	$\frac{2}{45} \frac{m_\pi^2}{M_V^2}$
4	$\frac{1}{189}$	$\frac{1}{135} (m_\pi r_\pi)^2$	$\frac{2}{45} \frac{m_\pi^2}{M_V^2}$
5	$\frac{1}{135}$	$\frac{4}{45} \left(\frac{m_\pi}{F_\pi} \right)^2 (\alpha_9^r + \alpha_{10}^r)$	0
6	$\frac{1}{315}$	0	0
7	$\frac{1}{945}$	0	0

PRD 86:037502
(2012)



Charged Pion Contribution: χ PT

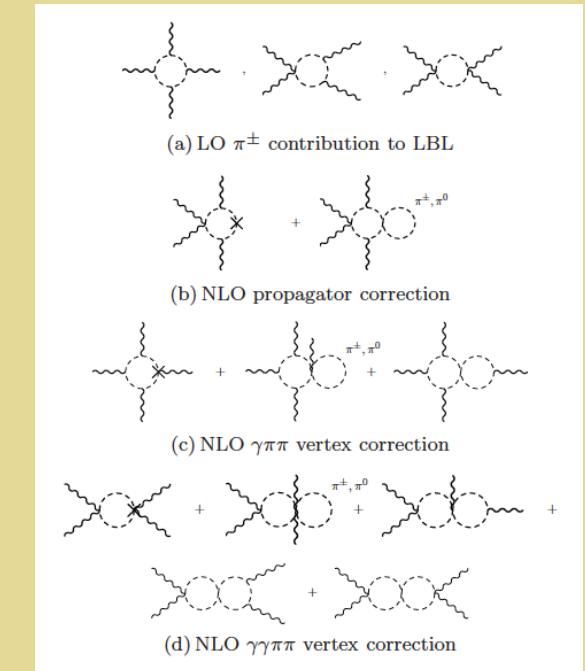
Kevin Engel (Caltech), Hiren Patel (Wisconsin), MRM

Beyond leading order: embedding
subgraphs in full HLBL contribution

Operator	1 loop χ PT	2 loop	VMD
$\mathcal{O}_1^{(8)}$	1/9	$\frac{m_\pi^2}{F_\pi^2} \frac{16}{3} (\alpha_9^r + \alpha_{10}^r)$	0
$\mathcal{O}_2^{(8)}$	1/45	0	0

Pol'ability

n	1 loop	2 loop	VMD
1	$\frac{1}{45}$	$\frac{1}{3} \left\{ \frac{1}{9} (m_\pi r_\pi)^2 + \frac{4}{5} \left(\frac{m_\pi}{F_\pi} \right)^2 (\alpha_9^r + \alpha_{10}^r) \right\}$	$\frac{2}{9} \frac{m_\pi^2}{M_V^2}$
2	$\frac{2}{45}$	$\frac{1}{9} \left\{ \frac{1}{3} (m_\pi r_\pi)^2 + \frac{1}{2} \frac{m_\pi^2}{\Lambda_\chi^2} + \frac{44}{5} \left(\frac{m_\pi}{F_\pi} \right)^2 (\alpha_9^r + \alpha_{10}^r) \right\}$	$\frac{2}{9} \frac{m_\pi^2}{M_V^2}$
3	$\frac{2}{315}$	$\frac{1}{135} (m_\pi r_\pi)^2$	$\frac{2}{45} \frac{m_\pi^2}{M_V^2}$
4	$\frac{1}{189}$	$\frac{1}{135} (m_\pi r_\pi)^2$	$\frac{2}{45} \frac{m_\pi^2}{M_V^2}$
5	$\frac{1}{135}$	$\frac{4}{45} \left(\frac{m_\pi}{F_\pi} \right)^2 (\alpha_9^r + \alpha_{10}^r)$	0
6	$\frac{1}{315}$	0	0
7	$\frac{1}{945}$	0	0



Charge
radius

Charged Pion Contribution: χ PT

Kevin Engel (Caltech), Hiren Patel (Wisconsin), MRM

Beyond leading order: embedding
subgraphs in full HLBL contribution

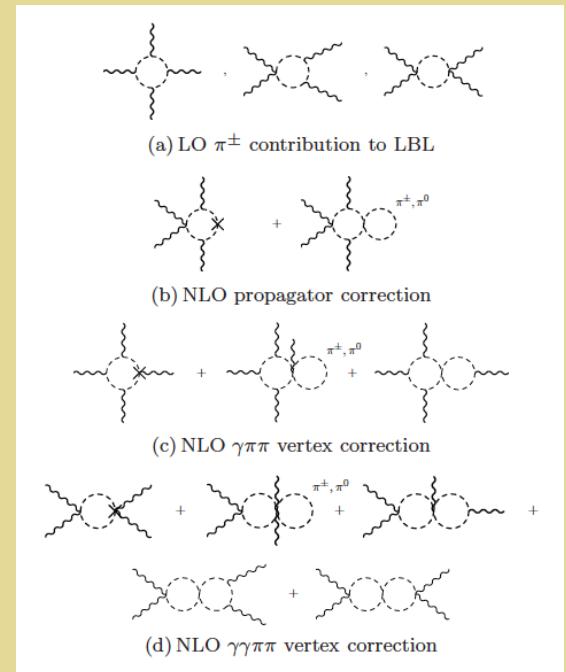
Operator	1 loop χ PT	2 loop	VMD
$\mathcal{O}_1^{(8)}$	1/9	$\frac{m_\pi^2}{F_\pi^2} \frac{16}{3} (\alpha_9^r + \alpha_{10}^r)$	0
$\mathcal{O}_2^{(8)}$	1/45	0	0

Currently
Omitted

Pol'ability

n	1 loop	2 loop	VMD
1	$\frac{1}{45}$	$\frac{1}{3} \left\{ \frac{1}{9} (m_\pi r_\pi)^2 + \frac{4}{5} \left(\frac{m_\pi}{F_\pi} \right)^2 (\alpha_9^r + \alpha_{10}^r) \right\}$	$\frac{2}{9} \frac{m_\pi^2}{M_V^2}$
2	$\frac{2}{45}$	$\frac{1}{9} \left\{ \frac{1}{3} (m_\pi r_\pi)^2 + \frac{1}{2} \frac{m_\pi^2}{\Lambda_\chi^2} + \frac{44}{5} \left(\frac{m_\pi}{F_\pi} \right)^2 (\alpha_9^r + \alpha_{10}^r) \right\}$	$\frac{2}{9} \frac{m_\pi^2}{M_V^2}$
3	$\frac{2}{315}$	$\frac{1}{135} (m_\pi r_\pi)^2$	$\frac{2}{45} \frac{m_\pi^2}{M_V^2}$
4	$\frac{1}{189}$	$\frac{1}{135} (m_\pi r_\pi)^2$	$\frac{2}{45} \frac{m_\pi^2}{M_V^2}$
5	$\frac{1}{135}$	$\frac{4}{45} \left(\frac{m_\pi}{F_\pi} \right)^2 (\alpha_9^r + \alpha_{10}^r)$	0
6	$\frac{1}{315}$	0	0
7	$\frac{1}{945}$	0	0

PRD 86:037502
(2012)

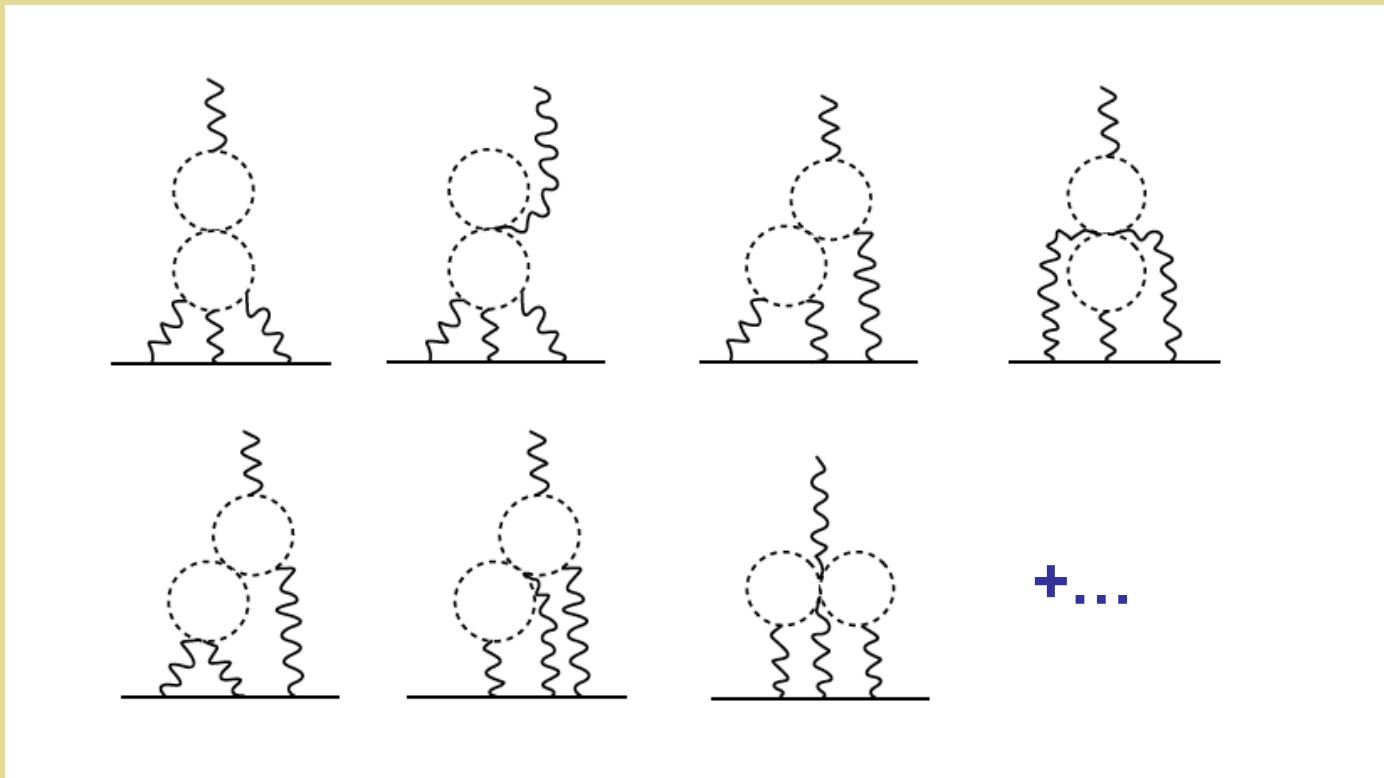


Charge
radius

Charged Pion Contribution: χ PT

Kevin Engel (Caltech), Hiren Patel (Wisconsin), MRM

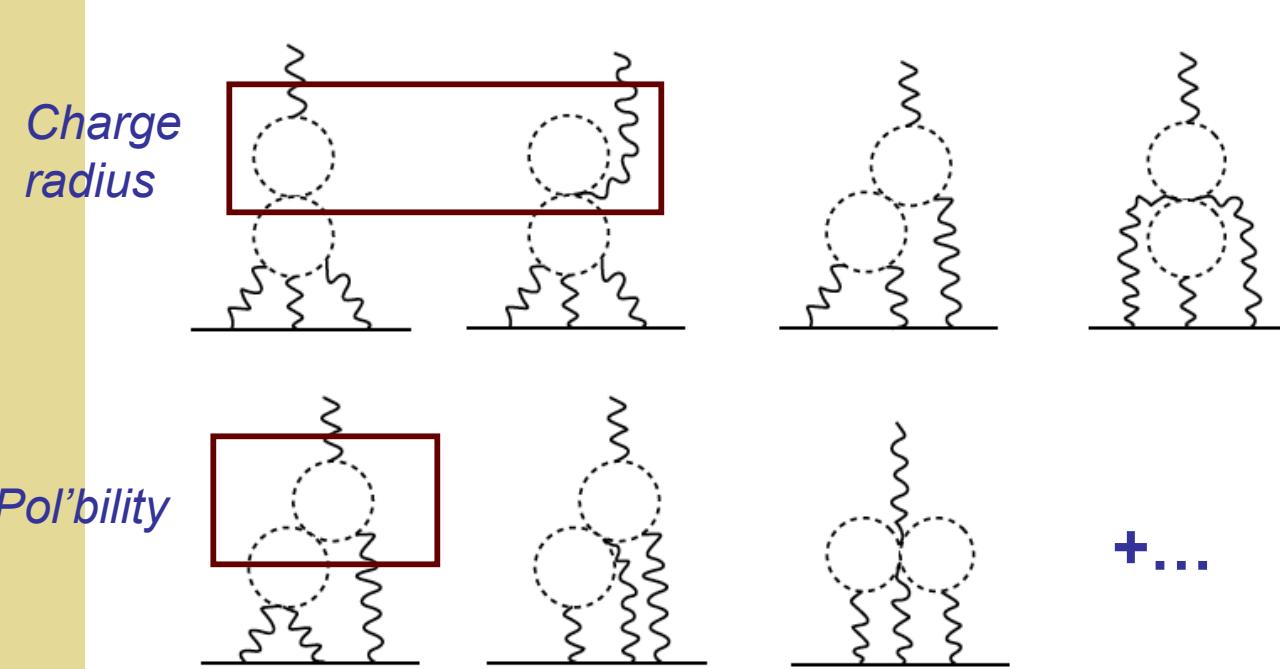
Beyond leading order: embedding
subgraphs in full HLBL contribution



Charged Pion Contribution: χ PT

Kevin Engel (Caltech), Hiren Patel (Wisconsin), MRM

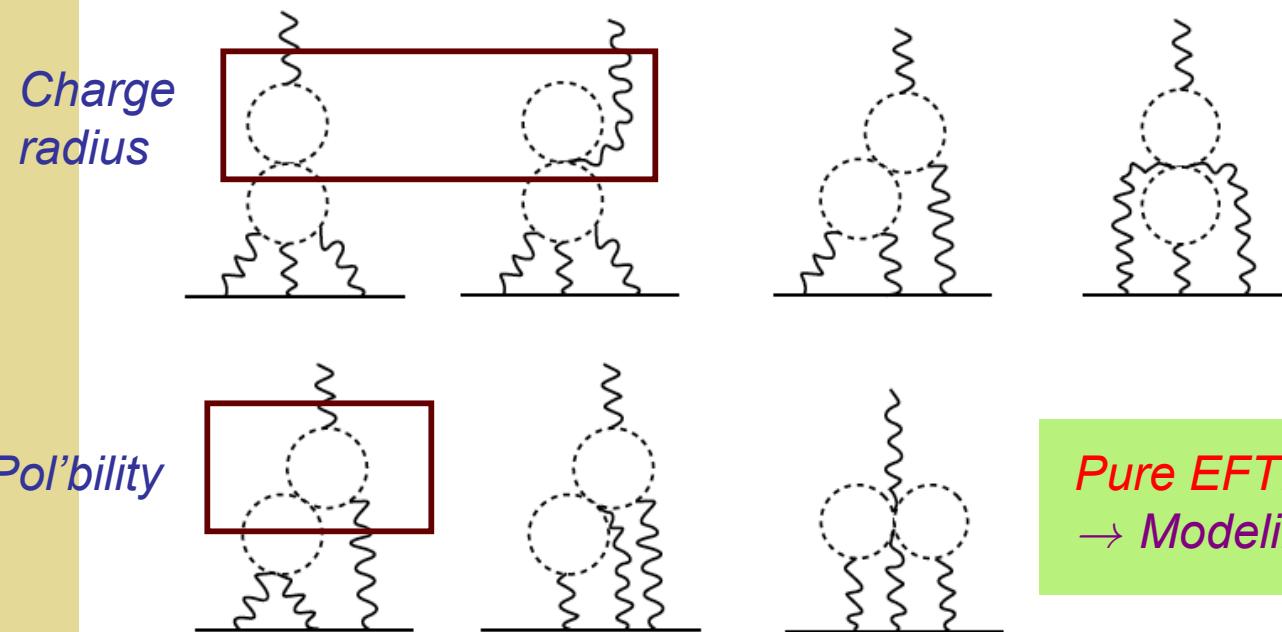
Beyond leading order: embedding
subgraphs in full HLBL contribution



Charged Pion Contribution: χ PT

Kevin Engel (Caltech), Hiren Patel (Wisconsin), MRM

Beyond leading order: embedding
subgraphs in full HLBL contribution

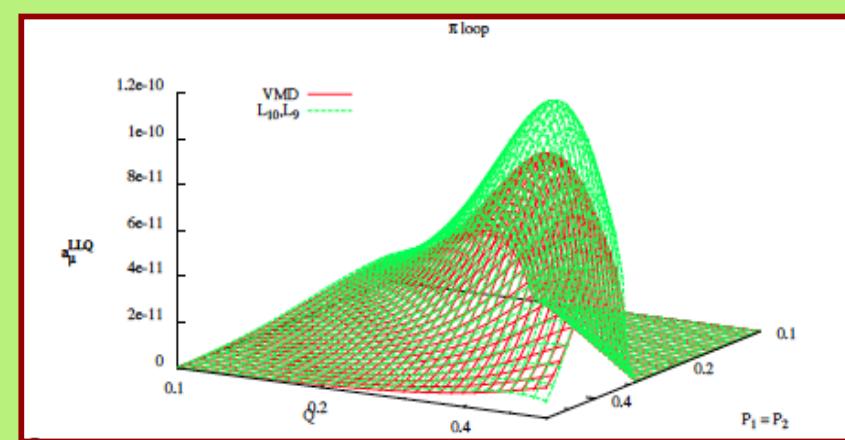
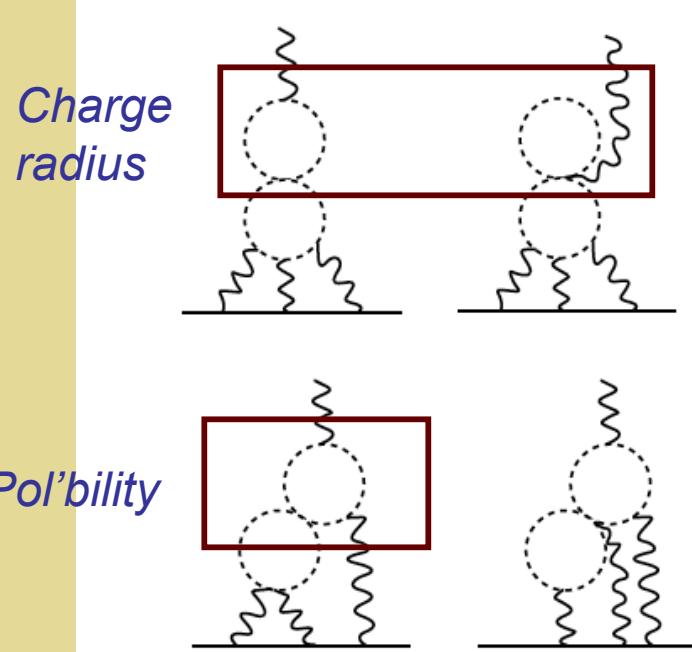


Pure EFT: Divergent
→ Modeling required

Charged Pion Contribution: χ PT

Kevin Engel (Caltech), Hiren Patel (Wisconsin), MRM

Beyond leading order: embedding
subgraphs in full HLBL contribution



Bijnens & Abyaneh 1208.3548 :
Include $\alpha_9 + \alpha_{10}$ to $k_{loop} \sim 500$ MeV
→ 10% increase in a_μ (π loop)

Beyond χ PT: Modeling High Q^2

Analogous problem: Pseudoscalar EM mass splitting

Donoghue, Holstein, Wyler '93

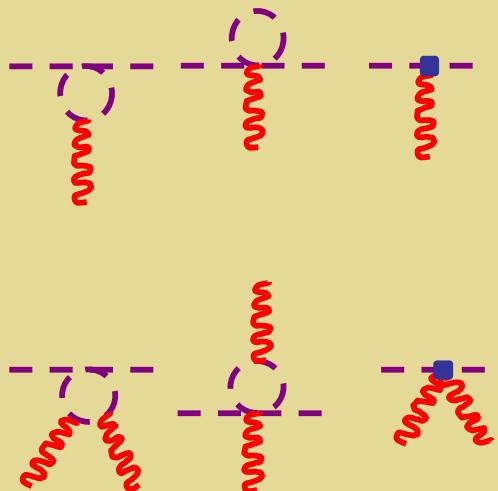
$$\begin{aligned}\Delta m_\pi^2 &= \delta m_+^2 - \delta m_0^2 \\ &= \frac{e^2}{2} \int \frac{d^4 q}{(2\pi)^4} \frac{1}{q^2} g^{\mu\nu} [T_{\mu\nu}^+(p, q) - T_{\mu\nu}^0(p, q)]\end{aligned}$$

Beyond χ PT: Modeling High Q^2

Analogous problem: Pseudoscalar EM mass splitting

Donoghue, Holstein, Wyler '93

$$\begin{aligned}\Delta m_\pi^2 &= \delta m_+^2 - \delta m_0^2 \\ &= \frac{e^2}{2} \int \frac{d^4 q}{(2\pi)^4} \frac{1}{q^2} g^{\mu\nu} [T_{\mu\nu}^+(p, q) - T_{\mu\nu}^0(p, q)]\end{aligned}$$



Pure EFT: Divergent
→ Modeling required

Beyond χ PT: Modeling High Q^2

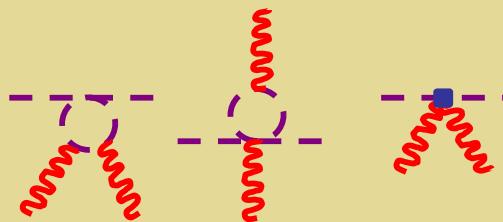
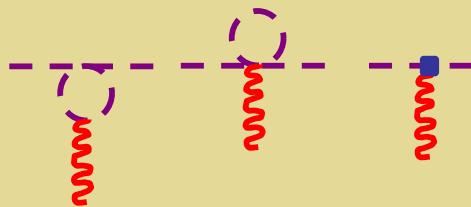
Analogous problem: Pseudoscalar EM mass splitting

Donoghue, Holstein, Wyler '93

$$\begin{aligned}\Delta m_\pi^2 &= \delta m_+^2 - \delta m_0^2 \\ &= \frac{e^2}{2} \int \frac{d^4 q}{(2\pi)^4} \frac{1}{q^2} g^{\mu\nu} [T_{\mu\nu}^+(p, q) - T_{\mu\nu}^0(p, q)]\end{aligned}$$

Quark counting rules

$$q^2 \rightarrow \infty : \quad T_{\mu\nu}(p, q) \sim \frac{1}{q^2}$$



Beyond χ PT: Modeling High Q^2

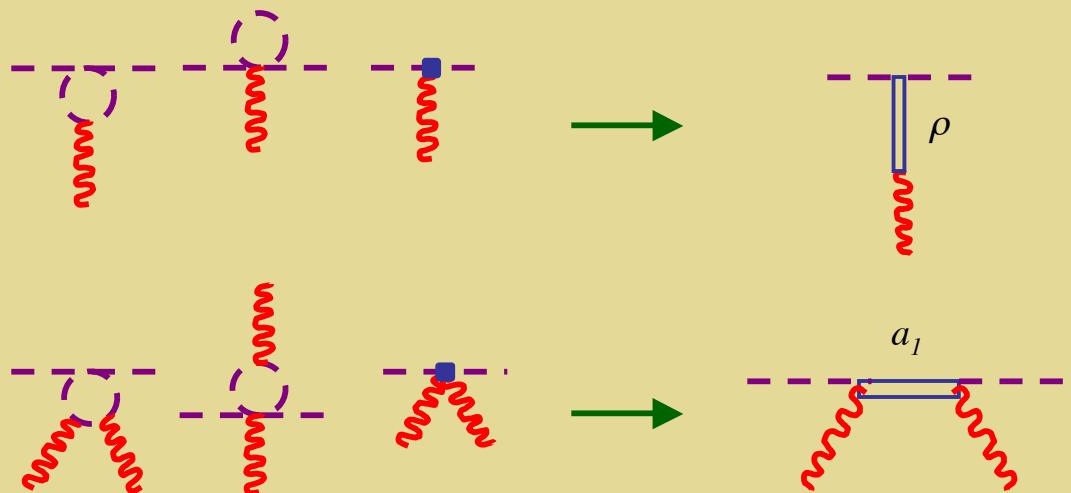
Analogous problem: Pseudoscalar EM mass splitting

Donoghue, Holstein, Wyler '93

$$\begin{aligned}\Delta m_\pi^2 &= \delta m_+^2 - \delta m_0^2 \\ &= \frac{e^2}{2} \int \frac{d^4 q}{(2\pi)^4} \frac{1}{q^2} g^{\mu\nu} [T_{\mu\nu}^+(p, q) - T_{\mu\nu}^0(p, q)]\end{aligned}$$

Quark counting rules

$$q^2 \rightarrow \infty : \quad T_{\mu\nu}(p, q) \sim \frac{1}{q^2}$$



$$\frac{1}{Q^2 + M_V^2} \rightarrow r_\pi^2 = \frac{6}{M_V^2}$$

$$\frac{1}{Q^2 + M_A^2} \rightarrow \alpha_9 + \alpha_{10} \sim \frac{1}{M_A^2}$$

Beyond χ PT: Modeling High Q^2

Analogous problem: Pseudoscalar EM mass splitting

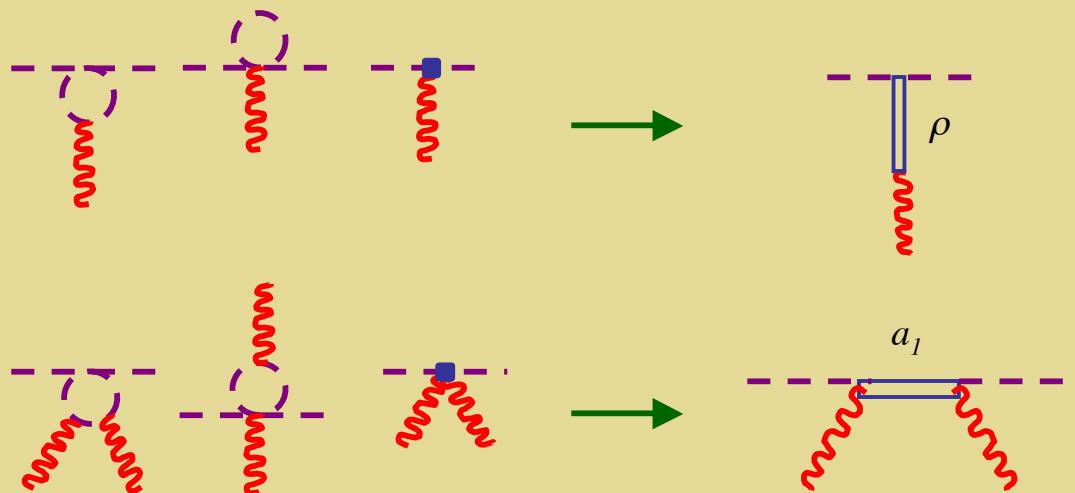
Donoghue, Holstein, Wyler '93

Model: $\Delta m_\pi^2 = 2 m_\pi \times 5.6 \text{ MeV}$

Expt: $\Delta m_\pi^2 = 2 m_\pi \times 4.6 \text{ MeV}$

Quark counting rules

$$q^2 \rightarrow \infty : \quad T_{\mu\nu}(p, q) \sim \frac{1}{q^2}$$

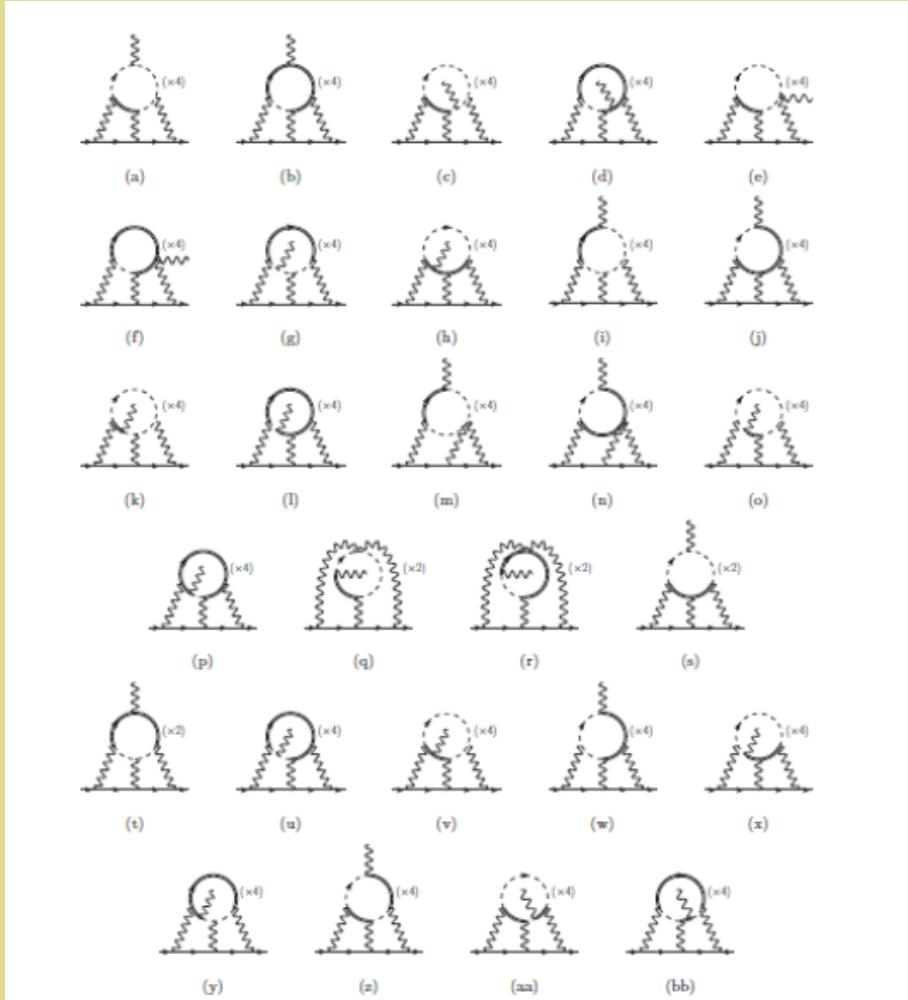


$$\frac{1}{Q^2 + M_V^2} \rightarrow r_\pi^2 = \frac{6}{M_V^2}$$

$$\frac{1}{Q^2 + M_A^2} \rightarrow \alpha_9 + \alpha_{10} \sim \frac{1}{M_A^2}$$

Charged Pion Contribution: Model

Kevin Engel (Caltech), MRM



+ ρ pole

Charged Pion Contribution: Model

Kevin Engel (Caltech), MRM

$$a_\mu^{LO} = -4.5 \times 10^{-10} \quad \rightarrow \quad -11.6 \times 10^{-10}$$

$$a_\mu^{VMD} = -1.7 \times 10^{-10} \quad \rightarrow \quad -6.7 \times 10^{-10}$$

$$a_\mu^{HLS} = -.4 \times 10^{-10} \quad \rightarrow \quad -1.6 \times 10^{-10}$$

Preliminary

Charged Pion Contribution: Model

Kevin Engel (Caltech), MRM

$$a_\mu^{LO} = -4.5 \times 10^{-10} \quad \rightarrow \quad -11.6 \times 10^{-10}$$

$$a_\mu^{VMD} = -1.7 \times 10^{-10} \quad \overset{\text{Preliminary}}{\rightarrow} \quad -6.7 \times 10^{-10}$$

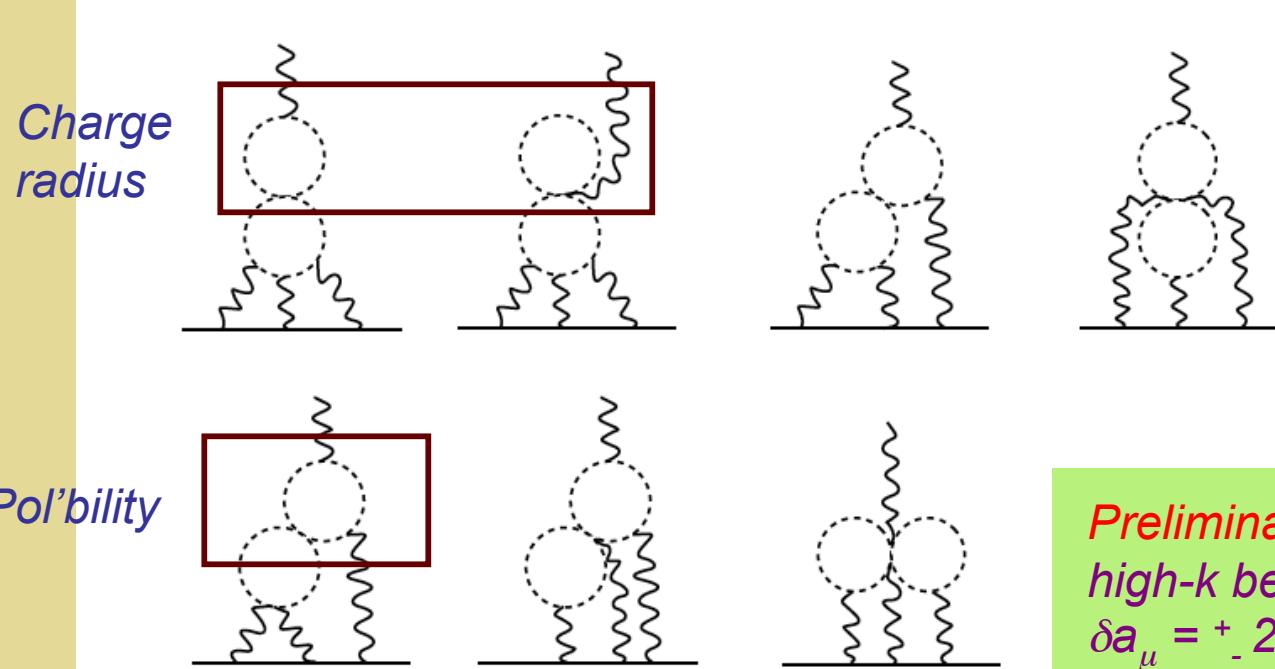
$$a_\mu^{HLS} = -.4 \times 10^{-10} \quad \rightarrow \quad -1.6 \times 10^{-10}$$

Model uncertainty ?

Charged Pion Contribution: χ PT

Kevin Engel (Caltech), Hiren Patel (Wisconsin), MRM

Beyond leading order: embedding
subgraphs in full HLBL contribution



Preliminary: modeling
high-k behavior:
 $\delta a_\mu = {}^{+}_{-} 20 \times 10^{-11}$

Summary & Outlook

HLBL: Compilation

Nyffeler 1001.3970

Contribution	BPP [8]	HKS, HK [9]	KN [10]	MV [11]	BP [5], MdRR [1]	PdRV [6]	N [13], JN [3]
π^0, η, η'	85±13	82.7±6.4	83±12	114±10	—	114±13	99±16
axial vectors	2.5±1.0	1.7±1.7	—	22±5	—	15±10	22±5
scalars	−6.8±2.0	—	—	—	—	−7±7	−7±2
π, K loops	−19±13	−4.5±8.1	—	—	—	−19±19	−19±13
π, K loops +subl. N_C	—	—	—	0±10	—	—	—
quark loops	21±3	9.7±11.1	—	—	—	2.3	21±3
Total	83±32	89.6±15.4	80±40	136±25	110±40	105±26	116±39

HLBL: Compilation

Nyffeler 1001.3970

Contribution	BPP [8]	HKS, HK [9]	KN [10]	MV [11]	BP [5], MdRR [1]	PdRV [6]	N [13], JN [3]
π^0, η, η'	85±13	82.7±6.4	83±12	114±10	—	114±13	99±16
axial vectors	2.5±1.0	1.7±1.7	—	22±5	—	15±10	22±5
scalars	-6.8±2.0	—	—	—	—	-7±7	-7±2
π, K loops	-19±13	-4.5±8.1	—	—	—	-19±19	-19±13
π, K loops +subl. N_C	—	—	—	0±10	—	—	—
quark loops	21±3	9.7±11.1	—	—	—	2.3	21±3
Total	83±32	89.6±15.4	80±40	136±25	110±40	105±26	116±39

HLBL: Compilation

Nyffeler 1001.3970

Contribution	BPP [8]	HKS, HK [9]	KN [10]	MV [11]	BP [5], MdRR [1]	PdRV [6]	N [13], JN [3]
π^0, η, η'	85±13	82.7±6.4	83±12	114±10	—	114±13	99±16
axial vectors	2.5±1.0	1.7±1.7	—	22±5	—	15±10	22±5
scalars	-6.8±2.0	—	—	—	—	-7±7	-7±2
π, K loops	-19±13	-4.5±8.1	—	—	—	-19±19	-19±13
π, K loops +subl. N_C	—	—	—	0±10	—	—	—
quark loops	21±3	9.7±11.1	—	—	—	2.3	21±3
Total	83±32	89.6±15.4	80±40	136±25	110±40	105±26	116±39

HLS

HLBL: Compilation

Nyffeler 1001.3970

Contribution	BPP [8]	HKS, HK [9]	KN [10]	MV [11]	BP [5], MdRR [1]	PdRV [6]	N [13], JN [3]
π^0, η, η'	85±13	82.7±6.4	83±12	114±10	—	114±13	99±16
axial vectors	2.5±1.0	1.7±1.7	—	22±5	—	15±10	22±5
scalars	6.8±2.0	—	—	—	—	7±7	7±2
π, K loops	-19±13	-4.5±8.1	—	—	—	-19±19	-19±13
π, K loops +subl. N_C	—	—	—	0±10	—	—	—
quark loops	21±3	9.7±11.1	—	—	—	2.3	21±3
Total	83±32	89.6±15.4	80±40	136±25	110±40	105±26	116±39

ENJL/VMD HLS

ENJL/VMD

HLBL: Compilation

Nyffeler 1001.3970

Contribution	BPP [8]	HKS, HK [9]	KN [10]	MV [11]	BP [5], MdRR [1]	PdRV [6]	N [13], JN [3]
π^0, η, η'	85±13	82.7±6.4	83±12	114±10	—	114±13	99±16
axial vectors	2.5±1.0	1.7±1.7	—	22±5	—	15±10	22±5
scalars	−6.8±2.0	—	—	—	—	−7±7	−7±2
π, K loops	−19±13	−4.5±8.1	—	—	—	−19±19	−19±13
π, K loops +subl. N_C	—	—	—	0±10	—	—	—
quark loops	21±3	9.7±11.1	—	—	—	2.3	21±3
Total	83±32	89.6±15.4	80±40	136±25	110±40	105±26	116±39

KTE, MRM -16 + 20 ? -67 + 20 ?

$$\begin{array}{r} \text{New Total ?} & +78 & +\underline{24} ? & +55 & +\underline{27} ? \end{array}$$

Increase Δa_μ by $\sim 1\sigma$

HLBL: Compilation

Nyffeler 1001.3970

Contribution	BPP [8]	HKS, HK [9]	KN [10]	MV [11]	BP [5], MdRR [1]	PdRV [6]	N [13], JN [3]
π^0, η, η'	85±13	82.7±6.4	83±12	114±10	—	114±13	99±16
axial vectors	2.5±1.0	1.7±1.7	—	22±5	—	15±10	22±5
scalars	-6.8±2.0	—	—	—	—	-7±7	-7±2
π, K loops	-19±13	-4.5±8.1	—	—	—	-19±19	-19±13
π, K loops +subl. N_C	—	—	—	0±10	—	—	—
quark loops	21±3	9.7±11.1	—	—	—	2.3	21±3
Total	83±32	89.6±15.4	80±40	136±25	110±40	105±26	116±39

KTE, MRM

-16 +_- 20 ?

-67 +_- 20 ?

New Total ?

+78 +_- 24 ?

+55 +_- 27 ?

Asymptopia



Increase Δa_μ by ~ 1 σ

HLBL: Compilation

Nyffeler 1001.3970

Contribution	BPP [8]	HKS, HK [9]	KN [10]	MV [11]	BP [5], MdRR [1]	PdRV [6]	N [13], JN [3]
π^0, η, η'	85±13	82.7±6.4	83±12	114±10	—	114±13	99±16
axial vectors	2.5±1.0	1.7±1.7	—	22±5	—	15±10	22±5
scalars	−6.8±2.0	—	—	—	—	−7±7	−7±2
π, K loops	−19±13	−4.5±8.1	—	—	—	−19±19	−19±13
π, K loops +subl. N_C	—	—	—	0±10	—	—	—
quark loops	21±3	9.7±11.1	—	—	—	2.3	21±3
Total	83±32	89.6±15.4	80±40	136±25	110±40	105±26	116±39

KTE, MRM -16 + 20 ? -67 + 20 ?

$$\begin{array}{r} \text{New Total ?} & +78 & +\underline{24} ? & +55 & +\underline{27} ? \end{array}$$

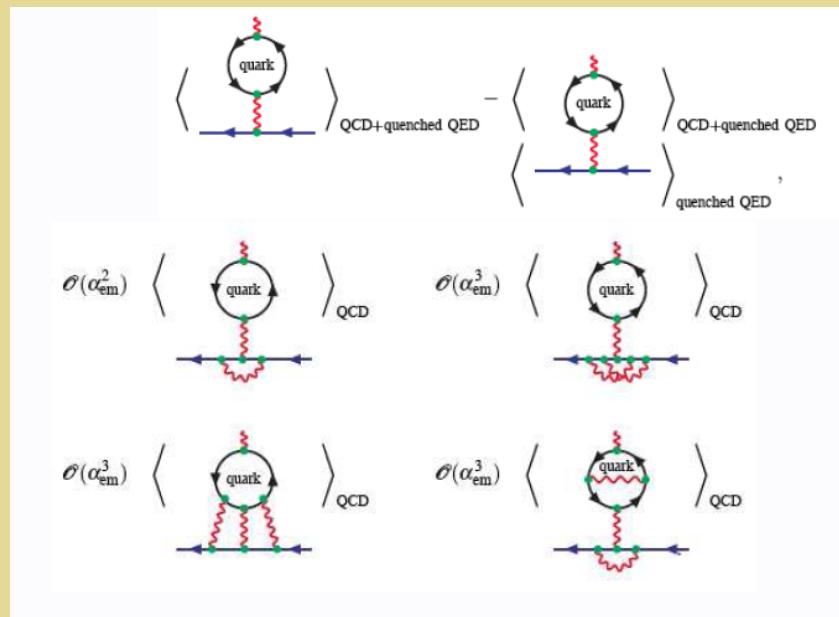
Asymptopia ✓ ✗

DSE Goeke, Fischer, Williams (B1, E2) +188 + 4 (stat)

Lattice QCD

See A. Juttner (B1)

- Blum, Izubuchi, ...: QED + QCD



- Rakow (QCDSF): 4pt function

Tenth Order QED

Aoyama, Hayakawa, Kinoshita, Nio '12

$$a_\mu(\text{QED}) = \sum_{n=1}^{\infty} \left(\frac{\alpha}{\pi} \right)^n a_\mu^{(2n)},$$

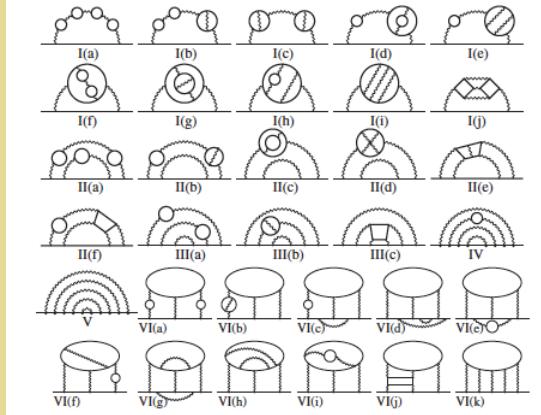


TABLE III. Contributions to muon $g - 2$ from QED perturbation term $a_\mu^{(2n)}(\alpha/\pi)^n \times 10^{11}$. They are evaluated with two values of the fine-structure constant determined by the Rb experiment and by the electron $g - 2$ (a_e).

order	with $\alpha^{-1}(\text{Rb})$	with $\alpha^{-1}(a_e)$
2	116 140 973.318 (77)	116 140 973.213 (30)
4	413 217.6291 (90)	413 217.6284 (89)
6	30 141.902 48 (41)	30 141.902 39 (40)
8	381.008 (19)	381.008 (19)
10	5.0938 (70)	5.0938 (70)
$a_\mu(\text{QED}) \times 10^{11}$	116 584 718.951 (80)	116 584 718.846 (37)

Summary

- *Hadronic contributions to a_μ remain an outstanding challenge for QCD theory*
- *Goal: reducing δa_μ^{TH} to below $\pm 15 \times 10^{-11}$ (future FNAL exp't goal) → a factor of 3-4 reduction in theory error*
- *Reaching this goal requires new scrutiny of all contributions previously considered “under control” (quark and pseudoscalar loops)*
- *Minimizing model-dependence → Checking consistency with chiral and pQCD limits*
- *Ultimate frontier: lattice QCD, but with insight from other approaches*